Electron Scattering on the Hoyle State and Carbon Production in Stars*
Maksym Chernykh

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**Motivation: Nuclear Structure**

- The Hoyle state is a prototype of $\alpha$-cluster states in light nuclei

- Some $\alpha$-cluster models predict the Hoyle state to consist of a dilute gas of weakly interacting $\alpha$ particles with properties of a Bose-Einstein Condensate (BEC)

- Comparison of high-precision electron scattering data with predictions of FMD and $\alpha$-cluster models

  Hoyle state cannot be understood as a true BEC

M. Chernykh et al., PRL 98 (2007) 032501
Motivation: Astrophysics

Triple alpha reaction rate

\[ r_{3\alpha} \propto \Gamma_{\text{rad}} \exp \left( -\frac{Q_{3\alpha}}{kT} \right) \]

\[ \Gamma_{\text{rad}} = \Gamma_\gamma + \Gamma_\pi = \frac{\Gamma_\gamma}{\Gamma} \cdot \frac{\Gamma_\pi}{\Gamma} \]

- Reaction rate with accuracy ±6% needed
- Total uncertainty \( \Delta r_{3\alpha}/r_{3\alpha} = \pm 12\% \) presently

S.M. Austin, NPA 758 (2005) 375c
Transition Form Factor to the Hoyle State

Extrapolation to zero momentum transfer

Fourier-Bessel analysis

Low-q data needed!

H. Crannell, data compilation (2005)
Low-q Extrapolation

\[ \sqrt{4\pi B(C^0, q)/q^2} = \frac{1}{6} (ME) \left[ 1 - \frac{q^2}{20} R_{tr}^2 + \ldots \right] \]

● **ME** = 5.37(22) fm², **R_{tr}** = 4.24(30) fm

● Large uncertainty because of narrow momentum transfer region

Low-q Extrapolation

\[ ME = 5.37(7) \text{ fm}^2, \quad R_{tr} = 4.30(12) \text{ fm} \]
Fourier-Bessel Analysis

- Transition form factor is the Fourier-Bessel transform of the transition charge density

\[ F'(q) = 4\pi \int_0^\infty \rho_{tr}(r) j_0(qr) r^2 \, dr \]

\[ \rho_{tr}(r) = \begin{cases} 
\sum_{\mu=1}^{\infty} a_{\mu} j_0(q_{\mu}r) & \text{for } r < R_c \\
0 & \text{for } r \geq R_c 
\end{cases} \]

with

\[ q_{\mu} = \frac{\mu \pi}{R_c} \]

- Data should be measured over a broad momentum transfer range
Fourier-Bessel Analysis

\[ q = 0.2 - 3.1 \text{ fm}^{-1} \]

\[ ME = 5.55(5) \text{ fm}^2 \]
Results

<table>
<thead>
<tr>
<th>Year</th>
<th>Analysis</th>
<th>Pair width</th>
<th>Ref.</th>
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<td>1967</td>
<td>PWBA</td>
<td></td>
<td>Crannell et al.</td>
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<tr>
<td>1970</td>
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<td>Strehl</td>
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<td>1970</td>
<td>Old average</td>
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<td>Ajzenberg-Selove</td>
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<td>Fourier-Bessel</td>
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<td>Crannell et al.</td>
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<td>2008</td>
<td>PWBA</td>
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\[ \Gamma_\pi = 62.2(10) \ \mu\text{eV} \]

\[ \text{Total uncertainty } \Delta r_{3\alpha}/r_{3\alpha} = \pm 10\% \]

\[ \text{Only } \Gamma_\pi'/\Gamma' \text{ needs still to be improved now} \]
Outlook

Theory systematically overpredicts experiment
Outlook

\[ ^{12}\text{C}: \, 0^+_3 \text{ and } 2^+_2 \text{ states} \]

\[ ^{16}\text{O}: \, 6\text{th excited } 0^+ \text{ state at } 15.1 \text{ MeV is the "Hoyle" state } \rightarrow ^{16}\text{O}(e,e'\alpha) \]